

On the Wilson loop in the dual representation within the dual Higgs model with dual Dirac strings

V. A. Ivanova^{*} and N. I. Troitskaya^{*}

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*Atominstitut der Österreichischen Universitäten, Arbeitsbereich Kernphysik und
Nukleare Astrophysik, Technische Universität Wien,
Wiedner Hauptstr. 8-10, A-1040 Wien, Österreich*

Abstract

The vacuum expectation value of the Wilson loop in the dual representation is calculated in the dual Higgs model with dual Dirac strings. It is shown that the averaged value of the Wilson loop in the dual representation obeys the area-law falloff. Quantum fluctuations of the dual-vector and the Higgs field around Abrikosov flux lines induced by dual Dirac strings in a dual superconducting vacuum and string shape fluctuations are taken into account.

^{*}Permanent Address: State Technical University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

1 Introduction

A dual $U(1)$ gauge Higgs model with dual Dirac strings suggested in Ref.[1] has been then applied to the calculation of the effective string energy caused by quantum field and string shape fluctuations around an Abrikosov flux line stretched between a quark and an antiquark [2,3]. In the symmetry broken phase due to a superconducting vacuum a dual Dirac string induces a dual-vector field $C_\mu(x; X)$ with a shape of an Abrikosov flux line. It has been shown that quantum field fluctuations give the contribution to the string tension [2], while the string shape fluctuations induce Coulomb-like contributions [4] with an universal coupling constant $\alpha_{\text{string}} = \pi/12$ and $\alpha_{\text{string}} = \pi/3$ for open and closed strings, respectively [3].

In this paper we turn to the investigation of the Wilson loop [5] in the dual representation. In the dual Higgs model with $U(1)$ gauge symmetry the Wilson loop is determined by

$$W(\mathcal{C}) = \exp \left\{ i g \oint_{\mathcal{C}} dx^\mu C_\mu(x) \right\}, \quad (1.1)$$

where g is a gauge coupling constant and \mathcal{C} is a closed contour.

In the symmetric phase the Lagrangian of the dual Higgs model with dual Dirac strings is defined by [1]

$$\begin{aligned} \mathcal{L}(x) = & \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + (\partial_\mu + ig C_\mu(x)) \Phi^*(x) (\partial^\mu - ig C^\mu(x)) \Phi(x) \\ & - \kappa^2 (v^2 - \Phi^*(x) \Phi(x))^2 + \mathcal{L}_{\text{free quark}}(x), \end{aligned} \quad (1.2)$$

where $\mathcal{L}_{\text{free quark}}(x)$ is the kinetic term of classical quarks and antiquarks [1].

Then $\Phi(x)$ is a complex Higgs field with a vacuum expectation value v , $\langle \Phi \rangle = v$, and κ is the coupling constant. The field strength $F^{\mu\nu}(x)$ is defined by: $F^{\mu\nu}(x) = \mathcal{E}^{\mu\nu}(x) - {}^*C^{\mu\nu}(x)$, where $C^{\mu\nu}(x) = \partial^\mu C^\nu(x) - \partial^\nu C^\mu(x)$, and ${}^*C^{\mu\nu}(x) = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} C_{\alpha\beta}(x)$ is a dual version of $C^{\mu\nu}(x)$ [1]. The electric field strength $\mathcal{E}^{\mu\nu}(x)$ is induced by a dual Dirac string and determined as follows [1–3]

$$\begin{aligned} \mathcal{E}^{\mu\nu}(x) &= Q \iint_{S(L)} d\sigma^{\mu\nu}(X) \delta^{(4)}(x - X) = \\ &= Q \iint_{S(L)} d\tau d\sigma \sigma^{\mu\nu}(X) \delta^{(4)}(x - X), \end{aligned} \quad (1.3)$$

where $S(L)$ is a 2-dimensional surface swept on the world-sheet by a shape L of a dual Dirac string and Q is the electric charge of a quark. The surface is parameterized by internal variables ($-\infty < \tau < +\infty$ and $0 \leq \sigma \leq \pi$) [1–3]:

$$d\sigma^{\mu\nu}(X) = \sigma^{\mu\nu}(X) d\tau d\sigma = \left(\frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma} - \frac{\partial X^\nu}{\partial \tau} \frac{\partial X^\mu}{\partial \sigma} \right) d\tau d\sigma, \quad (1.4)$$

such as $X^\mu(\tau, \sigma)|_{\sigma=0} = X_q^\mu(\tau)$, $X^\mu(\tau, \sigma)|_{\sigma=\pi} = X_{\bar{q}}^\mu(\tau)$ represent the world lines of an antiquark and a quark, respectively. As has been shown in [1] the inclusion of a dual Dirac string in terms of $\mathcal{E}^{\mu\nu}(x)$ defined by (1.3) saturates the electric Gauss law. Below in order to underscore that $\mathcal{E}^{\mu\nu}(x)$ is a functional of X , a point upon the surface $S(L)$, we introduce the notations $\mathcal{E}^{\mu\nu}(x) \rightarrow \mathcal{E}^{\mu\nu}(x; X)$ and $\mathcal{L}_{\text{free quark}}(x) \rightarrow \mathcal{L}_{\text{free quark}}(x; X)$.

In the symmetry–broken phase it is convenient to use a polar representation of the Higgs field Φ , i.e. $\Phi(x) = \rho(x) e^{i\vartheta(x)}$. In the polar representation of the Higgs field the Lagrangian (1.2) reads

$$\begin{aligned} \mathcal{L}(x) = & \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + (\partial_\mu + ig\tilde{C}_\mu(x))\rho(x)(\partial^\mu - ig\tilde{C}^\mu(x))\rho(x) - \\ & -\kappa^2(v^2 - \rho^2(x))^2 + \mathcal{L}_{\text{free quark}}(x; X), \end{aligned} \quad (1.5)$$

where we have denoted $\tilde{C}_\mu(x) = C_\mu(x) - \partial_\mu\vartheta(x)/g$. Below we omit a tilde above the C_μ –field.

The singular part of the phase field $\theta(x)$ and the electric tensor field $\mathcal{E}^{\mu\nu}(x; X)$ are related by [2]: ${}^*\mathcal{E}^{\mu\nu}(x; X) = (1/g)(\partial^\mu\partial^\nu - \partial^\nu\partial^\mu)\theta(x)$ (see also [6]).

The transition to the symmetry–broken phase can be performed by the shift of the ρ –field: $\rho(x) = v + \sigma(x)/\sqrt{2}$. This reduces the Lagrangian (1.5) to the form

$$\begin{aligned} \mathcal{L}(x) = & \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2} M_C^2 C_\mu(x) C^\mu(x) + \\ & + g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] C_\mu(x) C^\mu(x) + \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \\ & - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right]^2 + \mathcal{L}_{\text{free quark}}(x), \end{aligned} \quad (1.6)$$

where $M_C = \sqrt{2}gv$ and $M_\sigma = 2\kappa v$ are the masses of C_μ and σ fields, respectively.

2 The averaged value of the Wilson loop in the dual representation

In the symmetry–broken phase the dual–vector field C_μ fluctuates around an Abrikosov flux line induced by a dual Dirac string in a superconducting vacuum [1–3]. By varying the Lagrangian (1.6) with respect to the field $C_\nu(x)$ and using the constraint $\partial_\nu C^\nu(x) = 0$ we get the equation of motion [1–3]

$$(\square + M_C^2) C^\nu(x) = -\partial_\mu {}^*\mathcal{E}^{\mu\nu}(x; X) - 2g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] C^\nu(x). \quad (2.1)$$

In the tree approximation the vacuum expectation value $\langle C^\nu(x; X) \rangle$ obeys the equation [1–3]

$$(\square + M_C^2) \langle C^\nu(x; X) \rangle = -\partial_\mu {}^*\mathcal{E}^{\mu\nu}(x; X), \quad (2.2)$$

where we have taken into account that in the tree approximation $\langle \sigma(x) \rangle = 0$. The solution of Eq.(2.2) has the shape of a dual Abrikosov flux line and reads [1–3]

$$\langle C^\nu(x; X) \rangle = - \int d^4x' \Delta(x - x', M_C) \partial_\mu {}^*\mathcal{E}^{\mu\nu}(x'; X), \quad (2.3)$$

where $\Delta(x - y, M_C)$ is the Green function

$$\Delta(x - y, M_C) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{M_C^2 - k^2 - i0} e^{-ik \cdot (x - y)}. \quad (2.4)$$

The Abrikosov flux line provides a linearly rising interquark potential realizing confinement of electric quark charges [1–3].

The calculation of the averaged value of the Wilson loop $W(\mathcal{C})$ we perform integrating over the dual-vector field $c_\mu(x)$, $C_\mu(x) = \langle C_\mu(x; X) \rangle + c_\mu(x)$, and the Higgs field $\sigma(x)$ fluctuating around the Abrikosov flux line and obeying the constraints $\langle c_\mu(x) \rangle = 0$ and $\langle \sigma(x) \rangle = 0$ [2]. The fields $c_\mu(x)$ and $\sigma(x)$ are described by the Lagrangian

$$\begin{aligned} \mathcal{L}(x) &= \mathcal{L}_{\text{string}}(x; X) \\ &+ \frac{1}{2} c_\mu(x) \left\{ \left(\square + M_C^2 + g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] \right) g^{\mu\nu} - \partial^\mu \partial^\nu \right\} c_\nu(x) \\ &+ 2 g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] \langle C_\mu(x; X) \rangle c^\mu(x) \\ &+ g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] \langle C_\mu(x; X) \rangle \langle C^\mu(x; X) \rangle \\ &+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right]^2 + \mathcal{L}_{\text{free quark}}(x, X), \end{aligned} \quad (2.5)$$

which can be obtained from (1.6). In (2.5) we have used Eq.(2.2). The Lagrangian $\mathcal{L}_{\text{string}}(x; X)$ depends explicitly on the shape of a dual Dirac string and is defined by [1–3]

$$\int d^4x \mathcal{L}_{\text{string}}(x; X) = \frac{1}{4} M_C^2 \int \int d^4x d^4y \mathcal{E}_{\mu\alpha}(x; X) \Delta_\nu^\alpha(x - y, M_C) \mathcal{E}^{\mu\nu}(y; X), \quad (2.6)$$

where $\Delta_\nu^\alpha(x - y, M_C) = (g_\nu^\alpha + 2 \partial^\alpha \partial_\nu / M_C^2) \Delta(x - y; M_C)$.

Using Stokes' theorem and relations above the Wilson loop in the dual representation Eq.(1.1) can be recast into the form

$$\begin{aligned} W(\mathcal{C}) &= \exp \left\{ i \frac{1}{2} \int d^4x \mathcal{E}_{\mu\nu}(x; X) {}^* \mathcal{G}^{\mu\nu}(x) \right\} \exp \left\{ \frac{1}{2} i \int d^4x \langle C_{\mu\nu}(x; X) \rangle \mathcal{G}^{\mu\nu}(x) \right\} \\ &\times \exp \left\{ -i \int d^4x c_\nu(x) \partial_\beta \mathcal{G}^{\beta\alpha}(x) \right\}, \end{aligned} \quad (2.7)$$

where $\langle C_{\mu\nu}(x; X) \rangle = \partial_\mu \langle C_\nu(x; X) \rangle - \partial_\nu \langle C_\mu(x; X) \rangle$ and $\mathcal{G}^{\mu\nu}(x)$ is determined by

$$\mathcal{G}^{\mu\nu}(x) = g \iint_{S(\mathcal{C})} d\sigma^{\mu\nu}(Y) \delta^{(4)}(x - Y) = g \iint_{S(\mathcal{C})} d\tau' d\sigma' \sigma^{\mu\nu}(Y) \delta^{(4)}(x - Y). \quad (2.8)$$

A 2-dimensional surface $S(\mathcal{C})$ supports on the closed contour \mathcal{C} , $\partial S(\mathcal{C}) = \mathcal{C}$, and $Y^\alpha \equiv Y^\alpha(\tau', \sigma')$ is a point of this surface. One can show that the contribution of the first exponential is equal to unity, since the exponent

$$\frac{1}{2} \int d^4x \mathcal{E}_{\mu\nu}(x; X) {}^* \mathcal{G}^{\mu\nu}(x), \quad (2.9)$$

when the Dirac relation $Qg = 2\pi$ is used, amounts to either 2π or zero.

Averaging the Wilson loop $W(\mathcal{C})$ over fluctuations of the quantum fields $c_\mu(x)$ and $\sigma(x)$ we obtain the quantity depending only on string and quark–antiquark degrees of freedom $\langle W(\mathcal{C}) \rangle = W[X]$ defined by

$$W[X] = \int \mathcal{D}c_\mu \mathcal{D}\sigma W(\mathcal{C}) \exp \left\{ i \int d^4x \mathcal{L}(x; X) \right\}. \quad (2.10)$$

The Lagrangian $\mathcal{L}(x; X)$ is given by Eq.(2.2). The integral over the dual-vector field $c_\mu(x)$ is Gaussian and can be calculated explicitly. The result reads

$$\begin{aligned} W[X] &= \exp \left\{ i \int d^4x \left[\mathcal{L}_{\text{string}}(x; X) + \mathcal{L}_{\text{free quark}} + \frac{1}{2} i \langle C_{\mu\nu}(x; X) \rangle \mathcal{G}^{\mu\nu}(x) \right] \right\} \\ &\times \int \mathcal{D}\sigma \exp \left\{ i \int d^4x \mathcal{L}_{\text{eff}}(x; X) \right\}. \end{aligned} \quad (2.11)$$

The effective Lagrangian $\mathcal{L}_{\text{eff}}(x; X)$ amounts to

$$\begin{aligned} \int d^4x \mathcal{L}_{\text{eff}}(x; X) &= -i \frac{1}{2} \int d^4x \text{tr} \ln D^{-1}(x, y|\sigma)|_{y=x} \\ &+ \int d^4x \left\{ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right]^2 \right\} \\ &- \frac{1}{2} \iint d^4x d^4y \mathcal{J}^\mu(x; X) D_{\mu\nu}(x, y|\sigma) \mathcal{J}^\nu(y; X) \\ &+ \int d^4x g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] \langle C_\mu(x; X) \rangle \langle C^\mu(x; X) \rangle, \end{aligned} \quad (2.12)$$

where we have denoted

$$\mathcal{J}^\alpha(x; X) = \partial_\beta \mathcal{G}^{\beta\alpha}(x) - 2 g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] \langle C^\alpha(x; X) \rangle, \quad (2.13)$$

and $D_{\mu\nu}(x, y|\sigma)$ is the Green function of the c_μ -field in the external scalar field induced by the self-interactions of the Higgs field σ

$$i D_{\mu\nu}(x, y|\sigma) = \left\langle 0 \left| T \left(c_\mu(x) c_\nu(y) \exp i \int d^4z \mathcal{L}_{\text{int}}(z) \right) \right| 0 \right\rangle, \quad (2.14)$$

where T is a time-ordering operator and

$$\mathcal{L}_{\text{int}}(z) = g M_C \sigma(z) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(z)}{M_\sigma} \right] c_\mu(z) c^\mu(z). \quad (2.15)$$

Making all convolutions of the c_μ -fields we obtain $D_{\mu\nu}(x, y|\sigma)$ as a functional of the σ -field. Then, $\text{tr} \ln D^{-1}(x, y|\sigma)|_{y=x}$ in Eq.(2.12) means the trace over Lorentz indices of the inverse Green function of the $c_\mu(x)$ -field $D_{\mu\nu}^{-1}(x, y|\sigma)$ satisfying the equation

$$\begin{aligned} D_{\mu\nu}^{-1}(x, y|\sigma) &= \\ &= \left\{ \left(\square_x + M_C^2 + g M_C \sigma(x) \left[1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_\sigma} \right] \right) g_{\mu\nu} - \partial_\mu \partial_\nu \right\} \delta^{(4)}(x - y). \end{aligned} \quad (2.16)$$

The inverse Green function $D_{\mu\nu}^{-1}(x, y|\sigma)$ is connected with the Green function $D_{\mu\nu}(x, y|\sigma)$ via the relation: $\int d^4z D_{\mu\lambda}^{-1}(x, z|\sigma) D^\lambda{}_\nu(z, y|\sigma) = g_{\mu\nu} \delta^{(4)}(x - y)$.

Integration over the σ -field is a very complicated problem and cannot be carried out explicitly in a general form. In order to integrate out the σ -field we suggest to apply the approximation used in Refs. [1–3]: $M_\sigma \gg M_C$. This corresponds to a strong coupling limit of the Higgs fields $\kappa \gg g$. In this limit the scales of fluctuations of the σ -field are

small compared with M_σ and only one-loop contributions become dominant [1–3]. The result of the integration over the σ -field reads

$$\int \mathcal{D}\sigma \exp \left\{ i \int d^4x \mathcal{L}_{\text{eff}}(x; X) \right\} = \exp \left\{ i \int d^4x \delta \mathcal{L}_{\text{one-loop}}(x; X) \right\}, \quad (2.17)$$

where we have denoted

$$\begin{aligned} \int d^4x \delta \mathcal{L}_{\text{one-loop}}(x; X) &= -\frac{1}{2} g^2 i \Delta(0; M_\sigma) \int d^4x \langle C_\mu(x; X) \rangle \langle C^\mu(x; X) \rangle \\ &+ 2ig^2 M_C^2 \iint d^4x d^4y \langle C^\mu(x; X) \rangle \langle C^\nu(y; X) \rangle D_{\mu\nu}(x-y; M_C) \Delta(y-x; M_\sigma). \end{aligned} \quad (2.18)$$

Here $\Delta(x; M_\sigma)$ is the Green function of the free σ -field given by Eq.(2.4) by changing $M_C \rightarrow M_\sigma$, then $D_{\mu\nu}(x; M_C) = (g_{\mu\nu} + \partial_\mu \partial_\nu / M_C^2) \Delta(x; M_C)$ is the Green function of the free c_μ -field. We have also used the relations $M_C = \sqrt{2}gv$ and $M_\sigma = 2\kappa v$. Since $\langle C^\mu(x; X) \rangle$ is proportional to the electric charge of a quark Q , so due to the Dirac relation $Qg = 2\pi$ [1–3] the effective Lagrangian $\delta \mathcal{L}_{\text{one-loop}}(x; X)$ does not depend on Q . In turn the dependence of $\delta \mathcal{L}_{\text{one-loop}}(x; X)$ on the coupling constants g or κ enters implicitly via the masses M_C and M_σ . Using Eq.(2.18) we obtain the averaged value of the Wilson loop

$$W[X] = \exp \left\{ i \int d^4x \left[\mathcal{L}_{\text{eff string}}(x; X) + \mathcal{L}_{\text{free quark}}(x; X) + \frac{1}{2} \langle C_{\mu\nu}(x; X) \rangle \mathcal{G}^{\mu\nu}(x) \right] \right\}, \quad (2.19)$$

where $\mathcal{L}_{\text{eff string}}(x; X) = \mathcal{L}_{\text{string}}(x; X) + \delta \mathcal{L}_{\text{one-loop}}(x; X)$. The averaged value $W[X]$ given by Eq.(2.19) is a functional of a shape of a dual Dirac string and quark-antiquark degrees of freedom.

3 The averaged value of the Wilson loop for static quarks and strings

First, we suggest to consider the calculation of the r.h.s. of Eq.(2.19) for static quarks and strings. In this case $\mathcal{L}_{\text{free quark}}(x; X) = 0$ and the r.h.s. of Eq.(2.19) becomes equal to

$$W[X] = \exp \left\{ i \int d^4x \left[\mathcal{L}_{\text{eff string}}(x; X) + \frac{1}{2} \langle C_{\mu\nu}(x; X) \rangle \mathcal{G}^{\mu\nu}(x) \right] \right\}. \quad (3.1)$$

Following [1–3] we consider a static straight string of a length L stretched along the z -axis between a quark and an antiquark placed at the points $\vec{X}_Q = (0, 0, L/2)$ and $\vec{X}_{-Q} = (0, 0, -L/2)$, respectively. For such a static string the electric field strength $\mathcal{E}_{\mu\nu}(x; X)$ does not depend on time and is given by [1–3]

$$\vec{\mathcal{E}}(\vec{x}, \vec{X}) = \vec{e}_z Q \delta(x) \delta(y) \left[\theta(z - \frac{1}{2}L) - \theta(z + \frac{1}{2}L) \right], \quad (3.2)$$

where the unit vector \vec{e}_z is directed along the z -axis and $\theta(z)$ is the Heaviside function. This field strength produces the dual-vector potential

$$\langle \vec{C}(\vec{x}, \vec{X}) \rangle = -iQ \int \frac{d^3k}{4\pi^3} \frac{\vec{k} \times \vec{e}_z}{k_z} \frac{1}{M_C^2 + \vec{k}^2} \sin\left(\frac{k_z L}{2}\right) e^{i\vec{k} \cdot \vec{x}}. \quad (3.3)$$

In the Euclidean space-time the first term in the r.h.s. of Eq.(3.1) can be represented in the form

$$i \int d^4x \mathcal{L}_{\text{eff string}}(x; X) = -V(L) T, \quad (3.4)$$

where T is a time and $V(L)$ is the interquark potential induced by a dual Dirac string. Taking the limit $L \rightarrow \infty$, corresponding to an infinitely long string, and keeping only the terms linear in L we obtain [2]

$$\begin{aligned} V(L) &= \frac{Q^2 M_C^2}{8\pi} \left(1 + \frac{g^2}{16\pi^2} \frac{M_\sigma^2}{M_C^2} \right) \ell n \left(1 + \frac{M_\sigma^2}{M_C^2} \right) L = \\ &= \pi v^2 \left(1 + \frac{\kappa^2}{8\pi^2} \right) \ell n \left(1 + \frac{\kappa^2 Q^2}{2\pi^2} \right) L = \sigma L, \end{aligned} \quad (3.5)$$

where σ , the string tension, is determined

$$\sigma = \pi v^2 \left(1 + \frac{\kappa^2}{8\pi^2} \right) \ell n \left(1 + \frac{\kappa^2 Q^2}{2\pi^2} \right). \quad (3.6)$$

We have used here the relations $M_C = \sqrt{2}gv$, $M_\sigma = 2\kappa v$ and $Qg = 2\pi$.

The second term $\int d^4x \langle C_{\mu\nu}(x; X) \rangle \mathcal{G}^{\mu\nu}(x)$ in the exponent of (3.3) depends on the surface $S(\mathcal{C})$ bounded by the contour \mathcal{C} . One can show that for the shape of the string defined by Eq.(3.1) this term vanishes for an infinitely long contour \mathcal{C} embracing an infinitely large surface $S(\mathcal{C})$. Thus, the Wilson loop in the dual representation, calculated for static quarks, antiquarks and a static straight string stretched between them, acquires the form

$$W[X] = e^{-\sigma L T}, \quad (3.7)$$

where $L T$ is an area of a rectangular surface swept by a dual Dirac string of a length L . Such a behaviour of the averaged value of the Wilson loop given by Eq.(3.7) corresponds to the area-law falloff [5] testifying confinement of electric quark charges [5,6].

4 The averaged value of the Wilson loop for static quarks and fluctuating strings

A dynamics of a dual Dirac string we take into account considering string shape fluctuations for the string with static quarks and antiquarks attached to the ends. Following [3,4] the string shape fluctuations we define as $X^\mu = \bar{X}^\mu + \eta^\mu(\bar{X})$, where $\eta^\mu(\bar{X})$ describes fluctuations around the fixed surface $S(L)$ swept by the shape L and \bar{X}^μ is a point upon this surface. It is assumed that at the boundary $\partial S(L)$ of the surface $S(L)$ the fluctuating field $\eta^\mu(\bar{X})$ vanishes, i.e. $\eta^\mu(\bar{X})|_{\partial S(L)} = 0$ [3,4]. The Wilson loop averaged over the string shape fluctuations we define as $\langle\langle W(\mathcal{C}) \rangle\rangle = \langle W[X] \rangle$, where

$$\langle W[X] \rangle = \int \mathcal{D}\eta \exp \left\{ i \int d^4x \left[\mathcal{L}_{\text{eff string}}(x; \bar{X} + \eta) + \frac{1}{2} \langle C_{\mu\nu}(x; \bar{X} + \eta) \rangle \mathcal{G}^{\mu\nu}(x) \right] \right\}. \quad (4.1)$$

For the fluctuations around the static straight string of the length L stretched between static quark and antiquark along the z -axis the integration over η -fields gives at $L \rightarrow \infty$ [3]

$$\langle W[X] \rangle = e^{-\sigma L T - \delta V(L) T}, \quad (4.2)$$

where $\delta V(L) = -\alpha_{\text{string}}/L$ and α_{string} is the universal coupling constant equal to $\alpha_{\text{string}} = \pi/12$ and $\alpha_{\text{string}} = \pi/3$ for opened [3,4] and closed [3] strings, respectively.

5 Conclusion

We have calculated the averaged value of the Wilson loop in the dual representation defined for the dual Higgs model with dual Dirac strings with Abelian $U(1)$ gauge symmetry. We have shown that the averaged value of the Wilson loop in the dual representation obeys the area-law falloff testifying confinement of quarks and antiquarks. We have found that the main contribution to the area-law falloff comes from the quantum field fluctuations around the Abrikosov flux line induced by a dual Dirac string in the superconducting vacuum of the symmetry broken phase. The contribution of dual Dirac string shape fluctuations leads to the appearance of a Coulomb-like potential with a universal coupling constant equal to $\alpha_{\text{string}} = \pi/12$ and $\alpha_{\text{string}} = \pi/3$ for opened [3,4] and closed [3] strings, respectively.

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